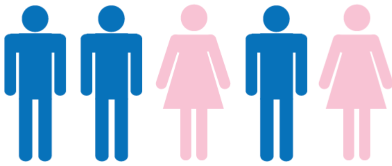
SUMMER QUIZ 2011
SOLUTIONS – PART 2

Medium 1

In the backward country of Genderdum, parents prefer sons to daughters. Each couple has up to two children; if the first child is a boy then they have no more children, and otherwise they have a second child. What is the ratio of boys to girls in Genderdum?



Answer: 50-50 (roughly).

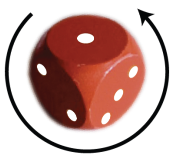
The chances of any given baby being a boy is 1/2. So, no matter how parents plan their families, the proportion of boys to girls in Genderdum will be about 50-50. (Of course, the exact proportion will depend upon the precise probabilities of boys and girls being born, random chance of who is born and who dies, and the like.)

Medium 2

Which number on the die is hidden directly beneath the skull?

Answer: 5.

Obviously, we can rule out 1 and 3. Also, the numbers on opposite sides of a die add to 7. So, 6 and 4 are also impossible, leaving only 2 and 5. However, on a regular die, the 1, 2, and 3 fit around a corner in a counterclockwise direction:



However, given the locations of the 1 and 3 on our die, this means that the 2 should be on the bottom. So, the 5 is directly underneath the skull.

Medium 3

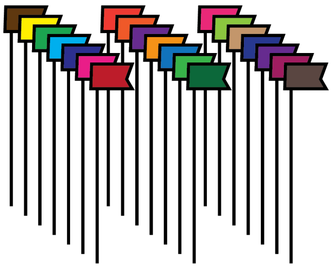
In the Olympic final, Larry finished in the middle of the runners. Curly finished in 10th place, somewhere behind Larry. Mo finished in 16th place. How many runners were in the race?

Answer: 17.

Larry being in the middle indicates there must be an odd number of runners. Then, since Curly is in 10th place, the middle must be 9th, 8th, 7th or 6th place, giving totals of 17, 15, 13, and 11 runners, respectively. Since there is a 16th place, the only possibility left is that there are 17 runners.

Medium 4

In an episode of the TV show Survivor Thailand, the two tribes were given a challenge involving 21 flags. The tribes took turns removing the flags: on each turn they could remove their choice of one, two or three flags. The tribe to remove the last flag was declared the winner, which turned out to be the tribe that went second. Should they have won?



Answer: No.

The team going first can win by always arranging that a multiple of 4 flags is left. So, on their first move they should remove 1 flag, leaving 20 flags. Then, however many flags the second team removes (leaving 19, 18 or 17 flags), the first team can bring the total down to 16. At the next stage the first team brings the number of flags down to 12, then to 8, to 4, and finally to 0.

MEDIUM 5

Which of the statements below are true?

> **One of these statements is false.**
> **Two of these statements are false.**
> **Three of these statements are false.**
> **Four of these statements are false.**
> **Five of these statements are false.**

Answer: Either the fourth statement alone is true, or we have a paradox.

Suppose we interpret the fourth statement as "**Exactly** four of these statements are false", and similarly for the other statements. Then at most one of the statements can be true, and the others must be false: it follows that this fourth one is the true statement.

However, suppose we interpret the fourth statement as "**At least** four of these statements are false", and similarly for the other statements. Then things get weird.

With the "at least" interpretation, a statement being true implies that the statements above it are true as well. That shows the fourth and fifth statements must be false, since otherwise there are too many true statements, So, we have at least two false statements, and the first and second statements must be true.

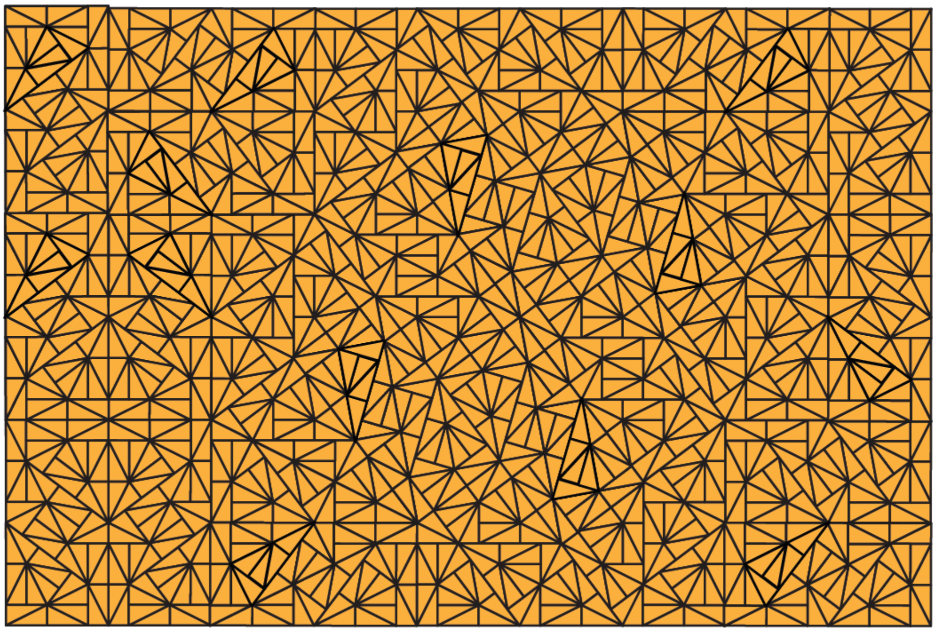
The curly question is, is the third statement true? If so, that says three statements are false, when in fact only the fourth and fifth statements are false. So, the third statement being true leads to a contradiction.

Therefore, it seems that the third statement must be false. But if so, then the third, fourth and fifth statements are all false. That means there are three false statements, which means that the third statement must in fact be true. Again, we have a contradiction!

It follows that, with the "at least" interpretation of the statements, we have an unavoidable paradox.

Medium 6

Below is one of the walls of Federation square. The tiles are all right-angled triangles, with shorter sides 60 centimeters and longer sides 120 centimeters. How many tiles have been used to cover the wall?



Answer: 1350.

Let's use a 60 cm side as our unit of length. Then the wall has dimensions 30 x 45, and so has area 1350 square units. As well, each triangle has area 1/2 x 2 x 1 = 1 square unit. So, there must be a total of 1350 triangles.

Medium 7

Wendy's sock drawer contains 4 blue socks, 4 red socks and 4 black socks. If Wendy pulls out three socks at random, what are the chances that there is a matching pair among the three?

Answer: 39/55.

Let's first figure out the chance that all three socks are different. The chances of drawing a red sock then a blue sock then a black sock are 4/12 x 4/11 x 4/10 = 8/165. There is the same chance of getting three different socks in any other order, and there are 3 x 2 x 1 = 6 different orders. It follows that the chance of getting three different socks is 6 x 8/165 = 16/55.

Now, the chance of getting a matching pair is exactly the chance of *not* getting three different socks, and so is just 1 – 16/55 = 39/55.

Medium 8

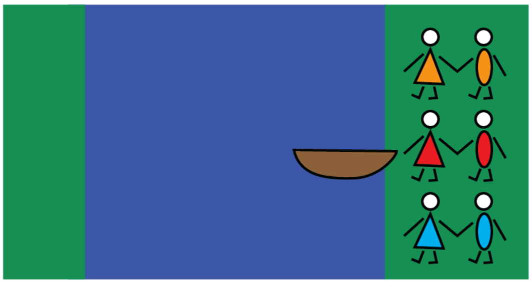
The Great Burk is juggling five balls, four red ones and a green one. On each beat of the music, he tosses a ball in the air, from one hand to the other; so, each ball is tossed on every 5th beat. You watch Burk for a minute, and in that minute you see him toss the green ball with his right hand 30 times. What is the total number of tosses that the Great Burk makes in a minute?

Answer: 300 (roughly).

If the green ball gets tossed by the right hand, then the next toss of this ball will be 5 beats later, by the left hand. So, the green ball is tossed by the right hand once every 10 beats, and that happened 30 times. So, the total number of tosses is about 30 x 10 = 300.

Medium 9

Three married couples arrive at a river. There is a boat they can use to cross the river, but it can only carry two people at a time. Also, no man is ever permitted to be alone with a woman other than his wife. Can they cross the river?



Answer: Yes.

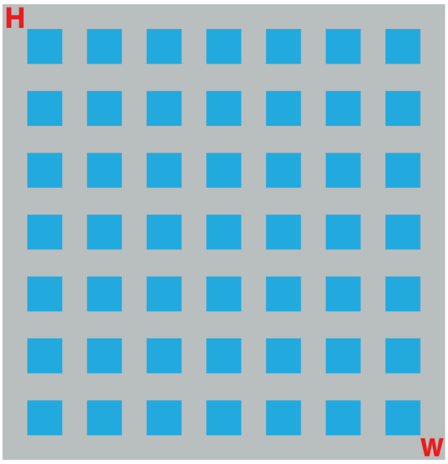
It turns out that they can all get to the other side of the river with nine crossings. Label the husbands as H1, H2, and H3, and similarly label the wives. Then the following table indicates who is on which side of the river at the beginning of each crossing. (The green and the red colourings indicate who is about to make the next crossing.)

one side of the river	the other side of the river
H1 H2 H3 W1 W2 W3	nobody
H1 H2 H3 W1 W3	W1 W2
H1 H2 H3 W2 W3	W1
H1 H3 W3	H2 W1 W2
H1 H2 H3 W3	W1 W2
H1 H2	H3 W1 W2 W3
H1 H2 H3	W1 W2 W3
H3	H1 H2 W1 W2 W3
H2 H3	H1 W1 W2 W3
nobody	H1 H2 H3 W1 W2 W3

It's also not hard to see that nine is the minimum possible number of crossings. There are six people who want to cross the river, which means at least three trips are required. However, each return of the boat requires a person to get it there. So, after four crossings in one direction and three returns (seven trips in all), there can only be five people on the desired side of the river; so, at least one more back-and-forth is required.

Medium 10

The city of Blueville consists of seven by seven blocks, and Angela's apartment and her workplace are at opposite corners of the city. What is the smallest number of blocks that Angela must travel in order to get from her apartment to work? How many different ways are there for her to make such a journey?



Answer: 14 blocks, and there are 3432 such routes.

Clearly, a shortest route will consist of walking 7 blocks down and 7 blocks to the right, a total of 14 blocks.

To count the number of ways to make a shortest walk, we'll label each walk down a block as D, and each walk of a block to the right as R. Then, any shortest walk consists of seven D's and seven R's, arranged in a sequence. We want to count how many ways there are to do that.

Imagine we have the 14 "words" D1, D2, …, D7, R1, R2, … R7 to arrange in a sequence. Then there are 14 x 13 x … 3 x 2 x 1 ways to do that: we write that number as 14! ("14 factorial"). Now, we think of D1 through to D7 as all standing for D, for walking down a block; so rearranging these words among themselves makes no difference to the actual route. There are 7! ways of rearranging D1 through to D7, and similarly there are 7! ways of arranging R1 through R7.

So, to obtain the number of shortest walks, we have to divide 14! (the number of arrangements of D1, D2, …, D7, R1, R2, … R7) by 7! x 7! (the number of ways of individually rearranging D1, D2, …, D7 and R1, R2, … R7).

Reaching for an unsmashed calculator, we find that the number of arrangements is 14! / (7! x 7 !) = 3432.